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Lecture 1 - Euler Characteristic

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A **polyhedron** is a 3D solid with straight edges and faces. (*Is a sphere a polyhedron?*) In a polyhedron, if two faces meet, they have a common edge between them and if two edges meet, they share a vertex. **Euler characteristic** or **Euler number** is a specific relation between the number of vertices (V), number of edges (E) and number of faces (F) for a polyhedron. (*What are the dimensions of a vertex, an edge and a face? Is there any connection between 2D polygons and 3D polyhedrons?*)

Count V, E, F for regular convex polyhedra, also called **platonic solids** (see Figure 1). (*What are regular convex polyhedra?*) If I tell you that there is a specific linear relation between V, E, and F that all of these platonic solids satisfy, can you solve three simultaneous equations and find out the relation? Assume the identity to be

$$xV + yE + zF = k,$$

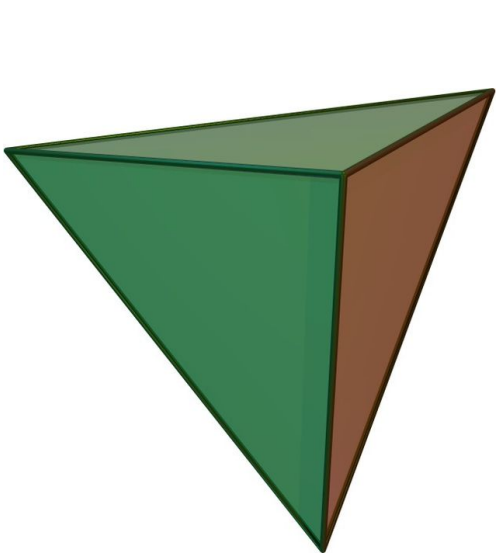
where x,y,z, and k are real numbers. Dividing the equation by x, we can rewrite it as

$$V + aE + bF = c.$$

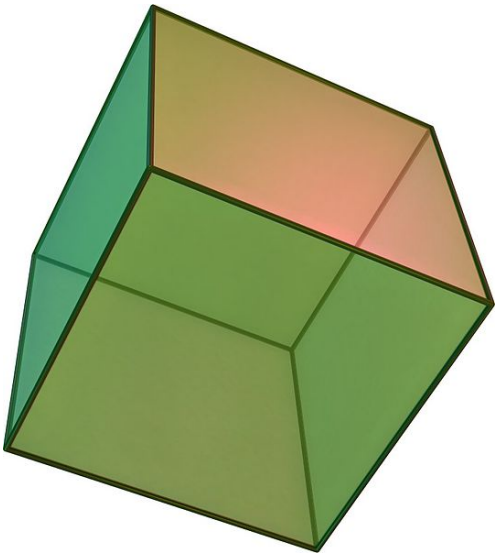
Find the values of a,b, and c and check whether all platonic solids satisfy this identity. In fact, all convex polyhedra should satisfy this identity (*Why?*), called **Euler's polyhedron formula**. In general, Euler characteristic,

$$\chi = V - E + F,$$

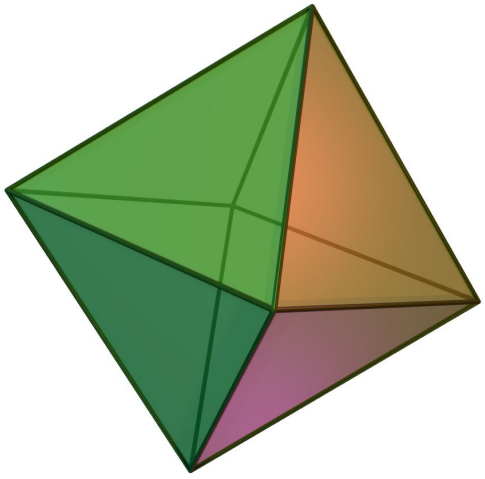
is an important quantity that describes the shape of a 3D object regardless of the way it is bent. For instance, we can prove that there are only five platonic solids using the value of χ.



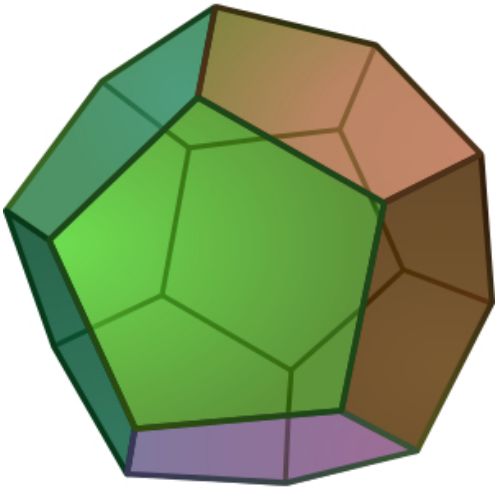
(a) Tetrahedron



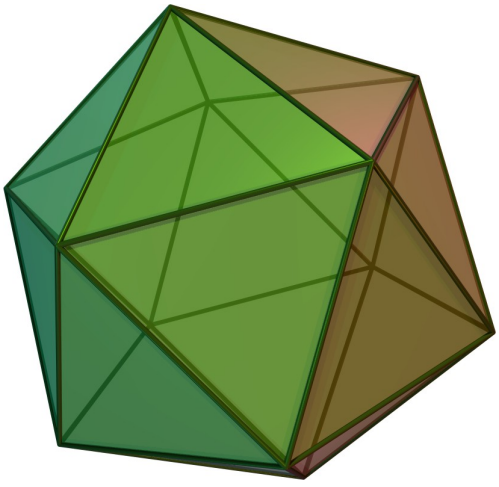
(b) Cube



(c) Octahedron



(d) Dodecahedron



(e) Icosahedron

Figure 1: Platonic Solids (Source: Wikipedia)